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Single-Mode Excited GHZ-type Entangled Coherent State

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Abstract A class of the single-mode excited GHZ-type entangled coherent states (EGHZECSs) are presented. we exhibit the remarkable properties of the single-mode EGHZECSs, depended on the excitation photon number, such as entanglement and nonlocality via investigating their concurrence of entanglement and examining their violation of CHSH inequality. Finally, we propose how to generate the EGHZECSs by using cavity QED and quantum measurement and by using BBO crystal and single-photon detection technique, respectively.

Keywords Excited GHZ-type entangled coherent states · Concurrence of entanglement · CHSH inequality

1 Introduction

Quantum entanglement has rightly been the subject of much study as an essential resource for quantum communication and information processing such as quantum computation [1, 2], quantum teleportation [3, 4], quantum dense coding [5, 6], quantum cryptography [7, 8] and so on. A large class of entangled states violate Bell inequalities [9–12], which means that the existence of such states cannot be explained by any local theory. It has been known that there exist at least two different types of multipartite entanglement, namely, the Greenberger-Horne-Zeilinger-type (GHZ-type) [13] entanglement and the W-type [14] entanglement. These two types of states cannot be converted to each other by local operation and classical communication with nonzero success probability.

The original study for quantum information processing focused on the discrete-variable entangled states, such as the polarizations of a photon or the discrete levels of an atom.

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In recent years, because the continuous-variable approach promises to be more compact and more efficient in both coding and manipulating quantum information, the continuous-variable entangled states have been developed rapidly from both theoretical and experimental point of view [15]. The coherent state, the simplest continuous-variable state, can be used to encode quantum information on continuous variables [16]. The code is based on single-mode coherent state superpositions (CSSs), the so called "cat states" [17, 18]. Entangled coherent state (ECS) [19] is normally regarded as a two-mode continuous-variable state. There have been a number of studies of ECS's revealing their quantum nonlocality [20, 21]. ECSs have found useful to perform some tasks in quantum information processing [22–25]. Gerry et al. [26] propose a method for generating ECSs of a two-mode field. Moreover, ones have studied the optimal quantum information processing via GHZ-type [27] and W-type ECSs [28]. Jeong's group [27] present the generation of GHZ-type ECSs using beam splitters (BSs) with a single-mode CSS and demonstrate Bell inequality violations for GHZ-type ECSs. Some theoretical schemes have been proposed to generate the GHZ-type ECS in cavity fields [29, 30].

On the other hand, based on another continuous-variable state, excited coherent state [31–33], the excited ECSs have also attracted much attention in the study of quantum entanglement. Kuang's group [34, 35] propose single-mode and two-mode excite ECSs and their generation via cavity QED. The optical generation of excited ECSs is also investigated by using the BSs and the type-I beta-barium borate (BBO) crystal [36, 37]. Very recently, Zhang and Xu [38] have examined the violation of Clauser-Horne-Shimony-Holt (CHSH) inequality for excited ECSs and propose a scheme of probabilistic teleportation via the excited ECSs.

In the present paper, enlightened by the above works, we will introduce another class of continuous-variable entangled states on the basis of GHZ-type ECSs [36, 37], called the single-mode excited GHZ-type entangled coherent states (EGHZECSs), which are obtained through actions of creation operator on the GHZ-type ECSs. We shall calculate the amount of entanglement for single-mode EGHZECSs by characterizing their concurrence of entanglement. The violation of the CHSH inequality for single-mode EGHZECSs is also evaluated. The organization of this paper is as follows. In Sect. 2, we present the definition of a class of the single-mode EGHZECSs under our consideration and give its Schmidt decomposition in term of the excited even(odd) coherent state. In Sect. 3, according to the definition of the concurrence, we calculate the concurrence of single-mode EGHZECSs and discuss the influence of the different excitation photon numbers on the concurrence. In Sect. 4, we examine the violation of CHSH inequality for single-mode EGHZECSs by using the formalism of Wigner representation in phase space based upon the parity measurement and the displacement operator. We devote Sect. 5 to propose some feasible schemes to generate single-mode EGHZECSs. In Sect. 6, we summarize our study and give a brief discussion on how to realize multi-mode excited case.

2 Single-Mode Excited GHZ-type Entangled Coherent States

In this section, we present the form of single-mode excited GHZ-type entangled coherent states. We first recall the three-mode GHZ-type ECSs as follows [27]

$$|\phi(\alpha,0)\rangle = N_0(|\alpha,\alpha,\alpha\rangle + e^{i\phi}|-\alpha,-\alpha,-\alpha\rangle), \tag{1}$$

where $|\alpha,\alpha,\alpha\rangle \equiv |\alpha\rangle_1 \otimes |\alpha\rangle_2 \otimes |\alpha\rangle_3$ with

$$|\alpha\rangle_i = \exp\left(-\frac{|\alpha|^2}{2} + \alpha a^\dagger\right)|0\rangle_i \tag{2}$$

being the usual coherent state and ϕ is the relative phase shift. The normalization constant is given by

$$N_0 = [2[1 + \exp(-6|\alpha|^2)\cos\phi]]^{-1/2}. \tag{3}$$

The EGHZECSs can be obtained by repeated application of the creation operator of a single-mode optical field on the three-mode GHZ-type ECSs, which are expressed as

$$|\phi(\alpha,m)\rangle = N_m a^{\dagger m}(|\alpha,\alpha,\alpha\rangle + e^{i\phi}|-\alpha,-\alpha,-\alpha\rangle), \tag{4}$$

where without any loss of generality we consider m-photon excitations of the mode 1 in the GHZ-type ECSs and N_m represents the normalization factor. Using the overlap $\langle\alpha|-\alpha\rangle = \exp(-2|\alpha|^2)$ and the following equality [39, 40]

$$a^n a^{\dagger m} = (-i)^{n+m} : H_{m,n}\left(ia^\dagger, ia\right) :, \tag{5}$$

where $: \; :$ represents the normal ordering for $(a^\dagger, a)$, $H_{m,n}(\eta,\eta^*)$ is the two-variable Hermite polynomial expressed as [41, 42]

$$\begin{aligned} H_{m,n}(\eta,\eta^*) &= \sum_{l=0}^{\min(m,n)} \frac{(-1)^l n!m!}{l!(m-l)!(n-l)!}\eta^{m-l}\eta^{*n-l} \\ &= \frac{\partial^{m+n}}{\partial t^m \partial t'^n}\exp\left(-tt' + t\eta + t'\eta^*\right)\Big|_{t=t'=0}, \end{aligned} \tag{6}$$

we can easily obtain

$$\langle\alpha|a^m a^{\dagger m}|\alpha\rangle = m!L_m(-|\alpha|^2), \qquad \langle\alpha|a^m a^{\dagger m}|-\alpha\rangle = m!e^{-2|\alpha|^2}L_m\left(|\alpha|^2\right), \tag{7}$$

and directly calculate the normalization factor

$$N_m = \left[2m!\left(L_m(-|\alpha|^2) + e^{-6|\alpha|^2}L_m(|\alpha|^2)\cos\phi\right)\right]^{-1/2}, \tag{8}$$

where $L_m(x)$ is the m-order Laguerre polynomial defined by [31]

$$L_m(x) = \sum_{l=0}^{m}\frac{(-1)^l m! x^l}{(l!)^2(m-l)!}. \tag{9}$$

It is quite clear that when $m = 0$, the states $|\phi(\alpha,m)\rangle$ reduce the usual GHZ-type ECSs in (1).

By considering $a^{\dagger m}|n\rangle = \sqrt{\frac{(n+m)!}{n!}}|n+m\rangle$, it is easily obtained the following expression

$$a^{\dagger m}|\alpha,\alpha,\alpha\rangle = e^{-|\alpha|^2}\sum_{r,s,t=0}^{\infty}\frac{\sqrt{(r+m)!}}{r!\sqrt{s!t!}}\alpha^{r+s+t}|r+m,s,t\rangle. \tag{10}$$

Further, $|\phi(\alpha, m)\rangle$ is expanded as in terms of Fock states

$$|\phi(\alpha,m)\rangle = N_m e^{-|\alpha|^2} \sum_{r,s,t=0}^{\infty} \frac{\sqrt{(r+m)!}}{r!\sqrt{s!t!}} \alpha^{r+s+t} \left[1 + e^{i\phi} (-1)^{r+s+t}\right] |r+m, s, t\rangle , \tag{11}$$

which implies that they are three truncations of the three-mode ESC given by equation with respect to the mode 1 in which all the terms related to the Fock states of the mode 1: $|0\rangle$, $|1\rangle, \ldots, |m-1\rangle$ are removed.

Due to the excited coherent states [31]

$$|\alpha, m\rangle = \left[m! L_m(-|\alpha|^2)\right]^{-1/2} a^{\dagger m} |\alpha\rangle , \tag{12}$$

which is intermediate between the Fock and the coherent states, it is not difficult to obtain the following form

$$|\phi(\alpha,m)\rangle = N_m \left[m! L_m(-|\alpha|^2)\right]^{1/2} \left[|\alpha, m\rangle |\alpha\rangle |\alpha\rangle + e^{i\phi} |-\alpha, m\rangle |-\alpha\rangle |-\alpha\rangle\right]. \tag{13}$$

Finally, considering the normalized even(odd) state [17, 18]

$$|\alpha\rangle_{\pm} = \left[2(1 \pm e^{-2|\alpha|^2})\right]^{-1/2} (|\alpha\rangle \pm |-\alpha\rangle) , \tag{14}$$

we have

$$\begin{aligned}
|\alpha,\alpha,\alpha\rangle + e^{i\phi}|-\alpha,-\alpha,-\alpha\rangle &= \left(1+e^{i\phi}\right)\left[(1+e^{-2|\alpha|^2})/2\right]^{3/2} |\alpha\rangle_+ |\alpha\rangle_+ |\alpha\rangle_+ \\
&\quad + \left(1-e^{i\phi}\right)\left[(1-e^{-2|\alpha|^2})/2\right]^{3/2} |\alpha\rangle_- |\alpha\rangle_- |\alpha\rangle_- \\
&\quad + \frac{(1-e^{i\phi})(1+e^{-2|\alpha|^2})[(1-e^{-2|\alpha|^2})/2]^{1/2}}{2} \\
&\quad \times \left[|\alpha\rangle_+ |\alpha\rangle_+ |\alpha\rangle_- + |\alpha\rangle_- |\alpha\rangle_+ |\alpha\rangle_+ + |\alpha\rangle_+ |\alpha\rangle_- |\alpha\rangle_+\right] \\
&\quad + \frac{(1+e^{i\phi})(1-e^{-2|\alpha|^2})[(1+e^{-2|\alpha|^2})/2]^{1/2}}{2} \\
&\quad \times \left[|\alpha\rangle_+ |\alpha\rangle_- |\alpha\rangle_- + |\alpha\rangle_- |\alpha\rangle_- |\alpha\rangle_+ + |\alpha\rangle_- |\alpha\rangle_+ |\alpha\rangle_-\right].
\end{aligned} \tag{15}$$

Substituting (15) into (4), after some algebra, we can get

$$\begin{aligned}
|\phi(\alpha,m)\rangle = N_m\sqrt{m!}e^{-\frac{|\alpha|^2}{2}} \Big\{ &\left(1+e^{i\phi}\right)(1+e^{-2|\alpha|^2})\sqrt{L_m^+(|\alpha|^2)}\, |\alpha,m\rangle_+ |\alpha\rangle_+ |\alpha\rangle_+ \\
&+ \left(1+e^{i\phi}\right)(1-e^{-2|\alpha|^2})\sqrt{L_m^+(|\alpha|^2)}\, |\alpha,m\rangle_+ |\alpha\rangle_- |\alpha\rangle_- \\
&+ \left(1-e^{i\phi}\right)(1-e^{-2|\alpha|^2})\sqrt{L_m^-(|\alpha|^2)}\, |\alpha,m\rangle_- |\alpha\rangle_- |\alpha\rangle_- \\
&+ \left(1-e^{i\phi}\right)(1+e^{-2|\alpha|^2})\sqrt{L_m^-(|\alpha|^2)}\, |\alpha,m\rangle_- |\alpha\rangle_+ |\alpha\rangle_+
\end{aligned}$$

$$+\left(1-e^{i\phi}\right)\sqrt{\left(1-e^{-4|\alpha|^2}\right)L_m^+(|\alpha|^2)}\left[|\alpha,m\rangle_+|\alpha\rangle_+|\alpha\rangle_-+|\alpha,m\rangle_+|\alpha\rangle_-|\alpha\rangle_+\right]$$

$$\left.+\left(1+e^{i\phi}\right)\sqrt{\left(1-e^{-4|\alpha|^2}\right)L_m^-(|\alpha|^2)}\left[|\alpha,m\rangle_-|\alpha\rangle_-|\alpha\rangle_++|\alpha,m\rangle_-|\alpha\rangle_+|\alpha\rangle_-\right]\right\} \tag{16}$$

where $|\alpha, m\rangle_\pm$ is the normalized excited even(odd) coherent state given by [43, 44],

$$|\alpha,m\rangle_\pm=\left[\frac{1\pm e^{-2|\alpha|^2}}{m!e^{-|\alpha|^2}L_m^\pm(|\alpha|^2)}\right]^{1/2}a^{\dagger m}|\alpha\rangle_\pm, \tag{17}$$

with the notation

$$L_m^\pm(x)=e^xL_m(-x)\pm e^{-x}L_m(x). \tag{18}$$

We call (16) the Schmidt decomposition [45] of the EGHZECSs $|\phi(\alpha,m)\rangle$ in the excited even(odd) coherent state.

3 Entanglement Properties of the EGHZECSs

In this section, we investigate the entanglement properties of EGHZECSs by analyzing the degree of entanglement. Many measures of entanglement have been introduced and analyzed [46–48]. Here we adopt the concurrence of entanglement [49–51] to characterize the entanglement of the EGHZECSs. The concurrence for a pure state $|\psi\rangle$ is defined by $C=|\langle\psi|\sigma_y\otimes\sigma_y|\psi^*\rangle|$, which equals to unit for a maximally entangled state.

For a general bipartite continuous entangled state

$$|\psi\rangle=\mu|A\rangle\otimes|B\rangle+\nu|E\rangle\otimes|F\rangle, \tag{19}$$

where μ and ν are complex numbers, $|A\rangle$ and $|E\rangle$ are normalized states of mode 1 and similarly $|B\rangle$ and $|F\rangle$ are normalized states of mode 2. By transforming continuous variable components to discrete orthogonal basis and making use of the Schmidt decomposition, one finds the concurrence of $|\psi\rangle$ in (19) is expressed as

$$C=\frac{2|\mu||\nu|\sqrt{(1-|p_1|^2)(1-|p_2|^2)}}{|\mu|^2+|\nu|^2+2\mathrm{Re}(\mu^*\upsilon p_1p_2^*)}, \tag{20}$$

where the two overlapping functions are defined by

$$p_1=\langle A|E\rangle, \qquad p_2=\langle B|F\rangle. \tag{21}$$

From (20), one case for a maximally entangled state $|\psi\rangle$, namely, $C=1$, is given by $\mu=-\nu$ and $p_1=p_2$.

From (13), we can see that the EGHZECSs $|\phi(\alpha,m)\rangle$ are two-component entangled state between mode 1 and modes 2 and 3. Thus, Due to (7) and (12), two overlapping functions are given by

$$p_1(\alpha,m)=\langle-\alpha,m|\,\alpha,m\rangle=\frac{\exp(-2|\alpha|^2)L_m(|\alpha|^2)}{L_m(-|\alpha|^2)}, \tag{22}$$

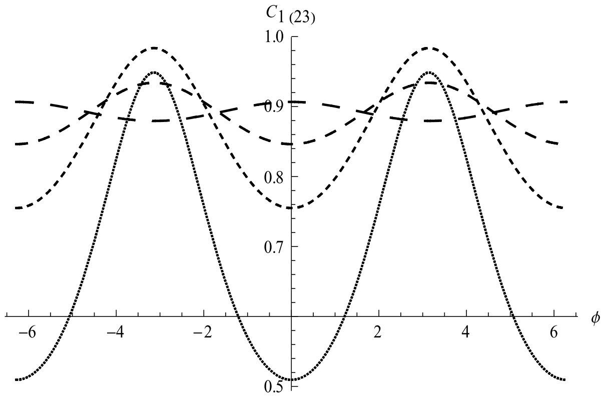


Fig. 1 Concurrence $C_{1(23)}$ of the EGHZECSs as a function of ϕ with $|\alpha|^2 = 0.2$ for the different m as follows: $m = 0$ (*solid line*), $m = 2$ (*dashed line*), $m = 4$ (*dotted line*), and $m = 9$ (*dash-dotted line*)

and

$$p_{23}(\alpha) = \langle -\alpha, -\alpha|\ \alpha, \alpha\rangle\ = \exp\left(-4\,|\alpha|^2\right). \tag{23}$$

Note that

$$0 < p_1(\alpha, m) \leq 1, \qquad p_1(\alpha, 0) = e^{-2|\alpha|^2}, \qquad p_1(\alpha, 1) = \frac{e^{-2|\alpha|^2}(1 - |\alpha|^2)}{1 + |\alpha|^2}. \tag{24}$$

According to (13) and (20), the concurrence of $|\phi(\alpha, m)\rangle$ between mode 1 and modes 2 and 3 is calculated by

$$\begin{aligned} C_{1(23)} &= \frac{\sqrt{(1 - |p_1(\alpha, m)|^2)(1 - |p_{23}(\alpha)|^2)}}{1 + p_1(\alpha, m)\,p_{23}(\alpha)\cos\phi} \\ &= \frac{\sqrt{[L_m^2(-|\alpha|^2) - \exp(-4|\alpha|^2)L_m^2(|\alpha|^2)][1 - \exp(-8|\alpha|^2)]}}{L_m(-|\alpha|^2) + \exp(-6|\alpha|^2)L_m(|\alpha|^2)\cos\phi}. \end{aligned} \tag{25}$$

Especially, when no excitations, i.e., $m = 0$, $\phi = 0$ and $\phi = \pi$, (25) recovers to

$$C^+_{1(23)} = \frac{\sqrt{[1 - \exp(-4|\alpha|^2)][1 - \exp(-8|\alpha|^2)]}}{1 + \exp(-6|\alpha|^2)}, \tag{26}$$

and

$$C^-_{1(23)} = \frac{\sqrt{[1 - \exp(-4|\alpha|^2)][1 - \exp(-8|\alpha|^2)]}}{1 - \exp(-6|\alpha|^2)}. \tag{27}$$

which is just the (31) of [52], respectively. In the limit $|\alpha| \to \infty$, the concurrence becomes 1 as we expected and when $|\alpha| \to 0$, $C^+_{1(23)} = 0$ and $C^-_{1(23)} = \frac{2\sqrt{2}}{3}$.

Similarly, the concurrence of $|\phi(\alpha, m)\rangle$ between mode 2 and modes 1 and 3 or between mode 3 and modes 1 and 2 is obtained as

$$C_{2(13)} = C_{3(12)} = \frac{\sqrt{[L_m^2(-|\alpha|^2) - \exp(-8|\alpha|^2)L_m^2(|\alpha|^2)][1 - \exp(-4|\alpha|^2)]}}{L_m(-|\alpha|^2) + \exp(-6|\alpha|^2)L_m(|\alpha|^2)\cos\phi}. \tag{28}$$

In order to observe the influence of the photon excitation on the EGHZECSs $|\phi(\alpha, m)\rangle$, we plot the concurrence $C_{1(23)}$ in (25) as a function of ϕ with $|\alpha|^2 = 0.2$ in Fig. 1 for the

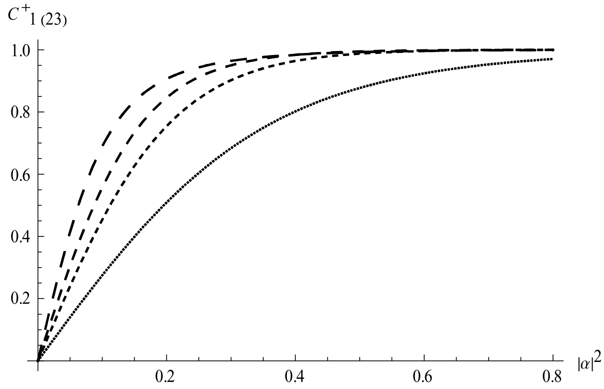


Fig. 2 Concurrence $C^{+}_{1(23)}$ ($\phi = 0$) of the EGHZECSs as a function of $|\alpha|^2$ for the different m as follows: $m = 0$ (*solid line*), $m = 2$ (*dashed line*), $m = 4$ (*dotted line*), and $m = 9$ (*dash-dotted line*)

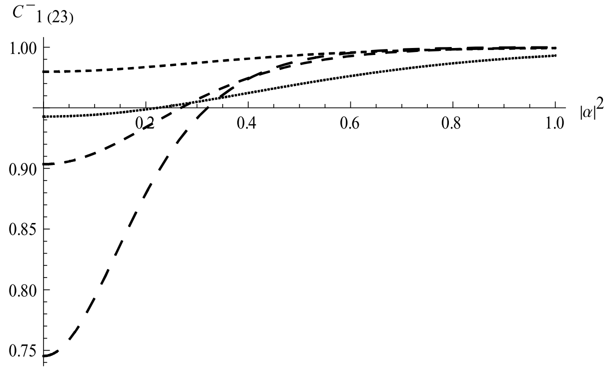


Fig. 3 Concurrence $C^{-}_{1(23)}$ ($\phi = \pi$) of the EGHZECSs as a function of $|\alpha|^2$ for the different m as follows: $m = 0$ (*solid line*), $m = 2$ (*dashed line*), $m = 4$ (*dotted line*), and $m = 9$ (*dash-dotted line*)

different values of m. We note that the concurrence $C_{1(23)}$ is sensitive to the phase ϕ and increases with the values of photon excitations m. In Figs. 2 and 3, the concurrence $C^{+}_{1(23)}$ ($\phi = 0$) and $C^{-}_{1(23)}$ ($\phi = \pi$) for the state $|\phi(\alpha, m)\rangle$ as a function of $|\alpha|^2$ is depicted for the different m. From Figs. 2 and 3, we easily see that when $|\alpha| \to 0$, $C^{+}_{1(23)} = 0$ and $C^{-}_{1(23)} = \frac{2\sqrt{2}}{3}$ for $m = 0$. It is shown that the concurrence increases with the increase of $|\alpha|^2$ for given parameter m. Especially, the concurrence tend to unit for the larger $|\alpha|^2$. These results can be confirmed analytically in the above discussion. In a word, we can increase the concurrence of the EGHZECSs by repeated application of the photon creation operator on the entangle coherent states.

4 CHSH Inequality Violation for the EGHZECSs

The quantum nonlocality for continue variable states has attracted much attention. ones have developed a Wigner function representation of the CHSH inequality using the parity operator as a quantum observable [27, 38, 53, 54]. In this section, we evaluate the violation of CHSH inequality for the EGHZECSs using the formalism of Wigner representation in phase space based upon the parity measurement and the displacement operator [55].

For this purpose, the observable $\Pi(z)$ is defined as

$$\Pi(z) = D(z)(-1)^N D^{\dagger}(z) = D(2z)(-1)^N, \tag{29}$$

where $D(z)=\exp[za^{\dagger}-z^{*}a]$ is the displacement operator with $D^{-1}(z)=D^{\dagger}(z)$, $N=a^{\dagger}a$ is Bosonic number operator, and $(-1)^{N}$ is the parity operator. Considering the coordinate eigenvector [56, 57]

$$|q\rangle=\pi^{-1/4}\exp\left[-\frac{q^{2}}{2}+\sqrt{2}qa^{\dagger}-\frac{1}{2}a^{\dagger}\right]|0\rangle, \tag{30}$$

and the normal product form of vacuum projector $|0\rangle\langle 0|=:\exp(-a^{\dagger}a):$ and using the operator identity

$$\exp\left(\lambda a^{\dagger}a\right)=:\exp\left[\left(e^{\lambda}-1\right)a^{\dagger}a\right]:, \tag{31}$$

we have the following operator identity via the IWOP technique [58]

$$\int dq\,|-q\rangle\langle q|=:\exp(-2a^{\dagger}a):=(-1)^{N}, \tag{32}$$

which is just a parity operator. Note that the eigenvalue of the observable $\Pi(z)$ is 1 when an even number of photons is detected and it is -1 when an odd number of photons is detected. For the three-mode case, the CHSH inequality based on the observable $\Pi(z)$ is expressed as [27]

$$\begin{aligned}BM=&\left|\langle\Pi(z_1)\Pi(z_2)\Pi(z_3)\rangle-\left\langle\Pi(z_1)\Pi(z_2')\Pi(z_3')\right\rangle-\left\langle\Pi(z_1')\Pi(z_2)\Pi(z_3')\right\rangle\right.\\&\left.-\left\langle\Pi(z_1')\Pi(z_2')\Pi(z_3)\right\rangle\right|\\&\le 2,\end{aligned} \tag{33}$$

where $\Pi(z_1)\Pi(z_2)\Pi(z_3)\equiv\Pi_1(z_1)\otimes\Pi_2(z_2)\otimes\Pi_3(z_3)$ and $\langle\rangle$ represents the expectation value.

Next, we recall the Wigner formalism parity via the IWOP technique. For a single-mode system, the Wigner operator $\Delta(z)$ in the coherent state $|\beta\rangle$ representation is expressed as [59, 60]

$$\Delta(z)=e^{2|z|^{2}}\int\frac{d^{2}\beta}{\pi^{2}}|\beta\rangle\langle-\beta|\exp\left[-2(\beta z^{*}-\beta^{*}z)\right]. \tag{34}$$

By using (2) and (32) and the IWOP technique, the Wigner operator in term of the parity operator is rewritten as

$$\Delta(z)=\frac{1}{\pi}e^{-2|z|^{2}+2a^{\dagger}z}:\exp\left[-2aa^{\dagger}\right]:e^{2az^{*}}=\frac{1}{\pi}D(z)(-1)^{N}D^{-1}(z). \tag{35}$$

So the three-mode Wigner function at a given phase point described by z_1, z_2 and z_3 is the expectation of the observable operator, i.e.,

$$W(z_1,z_2,z_3)=\langle\Delta_1(z_1)\Delta_2(z_2)\Delta_3(z_3)\rangle=\frac{1}{\pi^{3}}\langle\Pi(z_1)\Pi(z_2)\Pi(z_3)\rangle. \tag{36}$$

From (33) and (36), the Wigner representation of CHSH inequality is of the form

$$BM=\pi^{3}\left|W(z_1,z_2,z_3)-W(z_1,z_2',z_3')-W(z_1',z_2,z_3')-W(z_1',z_2',z_3)\right|\le 2. \tag{37}$$

In the following process, to evaluate the violation of CHSH inequality for the EGHZECSs $|\phi(\alpha,m)\rangle$, we first calculate the Wigner function, which is given by

$$\begin{aligned} W(z_1,z_2,z_3) &= \langle\phi(\alpha,m)|\,\Delta_1(z_1)\,\Delta_2(z_2)\,\Delta_3(z_3)\,|\phi(\alpha,m)\rangle \\ &= (N_m)^2\left[\langle\alpha,\alpha,\alpha| + e^{-i\phi}\langle-\alpha,-\alpha,-\alpha|\right]a^m\Delta_1(z_1)\,\Delta_2(z_2)\,\Delta_3(z_3) \\ &\quad\times a^{\dagger m}\left[|\alpha,\alpha,\alpha\rangle + e^{i\phi}|-\alpha,-\alpha,-\alpha\rangle\right]. \end{aligned} \tag{38}$$

Using (34) and $\langle\alpha|\beta\rangle = \exp(-\frac{|\alpha|^2}{2} - \frac{|\beta|^2}{2} + \alpha^*\beta)$ and the following integral formula

$$L_m(\xi\eta) = \frac{e^{\xi\eta}}{m!}\int\frac{d^2z}{\pi}z^m z^{*m}\exp\left[-|z|^2 + \xi z - \eta z^*\right], \tag{39}$$

we may obtain the following results

$$\begin{aligned} \langle\alpha|\,a^m\Delta_1(z_1)\,a^{\dagger m}\,|\alpha\rangle &= (-1)^m e^{2|z_1|^2-|\alpha|^2}\int\frac{d^2\beta}{\pi^2}\beta^m\beta^{*m}\exp\left[-|\beta|^2 + \left(\alpha^* - 2z_1^*\right)\beta\right. \\ &\quad\left. + \,(2z_1-\alpha)\,\beta^*\right] \\ &= \frac{(-1)^m\,m!e^{-2|\alpha-z_1|^2}}{\pi}L_m\left[|\alpha-2z_1|^2\right], \end{aligned} \tag{40}$$

$$\langle\alpha|\,a^m\Delta_1(z_1)\,a^{\dagger m}\,|-\alpha\rangle = \frac{(-1)^m\,m!e^{-2|z_1|^2-2\alpha z_1^*+2z_1\alpha^*}}{\pi}L_m\left[-(\alpha+2z_1)\left(\alpha^*-2z_1^*\right)\right], \tag{41}$$

$$\langle-\alpha|\,a^m\Delta_1(z_1)\,a^{\dagger m}\,|\alpha\rangle = \frac{(-1)^m\,m!e^{-2|z_1|^2+2\alpha z_1^*-2z_1\alpha^*}}{\pi}L_m\left[-\left(\alpha^*+2z_1^*\right)(\alpha-2z_1)\right], \tag{42}$$

and

$$\langle-\alpha|\,a^m\Delta_1(z_1)\,a^{\dagger m}\,|-\alpha\rangle = \frac{(-1)^m\,m!e^{-2|\alpha+z_1|^2}}{\pi}L_m\left[|\alpha+2z_1|^2\right]. \tag{43}$$

Therefore, putting (40)–(43) into (38), the Wigner function of $|\phi(\alpha,m)\rangle$ is simplified as

$$\begin{aligned} W(z_1,z_2,z_3) = \frac{(N_m)^2(-1)^m m!}{\pi^3}&\left\{f(\alpha,z_1,z_2,z_3)L_m\left[|\alpha-2z_1|^2\right]\right. \\ &+ f(-\alpha,z_1,z_2,z_3)L_m\left[|\alpha+2z_1|^2\right] \\ &+ e^{-i\phi}g(\alpha,z_1,z_2,z_3)L_m\left[-\left(\alpha^*+2z_1^*\right)(\alpha-2z_1)\right] \\ &\left.+ e^{i\phi}g(-\alpha,z_1,z_2,z_3)L_m\left[-(\alpha+2z_1)\left(\alpha^*-2z_1^*\right)\right]\right\}, \end{aligned} \tag{44}$$

where

$$f(\alpha,z_1,z_2,z_3) = \exp\left[-2|\alpha-z_1|^2 - 2|\alpha-z_2|^2 - 2|\alpha-z_3|^2\right], \tag{45}$$

and

$$\begin{aligned} &g(\alpha,z_1,z_2,z_3) \\ &\quad= \exp\left[-2|z_1|^2 + 2\alpha z_1^* - 2z_1\alpha^* - 2|z_2|^2 + 2\alpha z_2^* - 2z_2\alpha^* - 2|z_3|^2 + 2\alpha z_3^* - 2z_3\alpha^*\right]. \end{aligned} \tag{46}$$

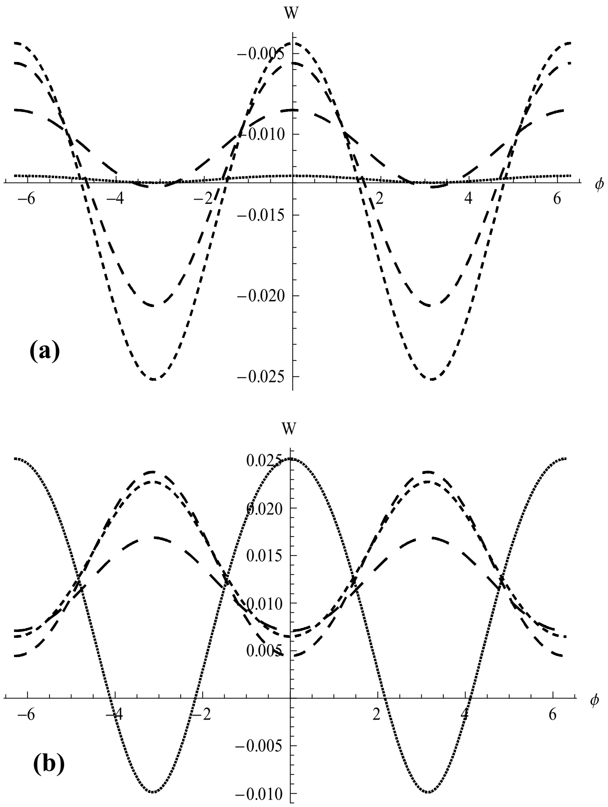


Fig. 4 Wigner function of the EGHZECSs as a function of ϕ with $|\alpha| = 0.3$, $|z_1| = 0.5$ and $|z_2| = |z_3| = 0.1$ for the different m as follows: (**a**) $m = 1$ (*solid line*), $m = 3$ (*dashed line*), $m = 5$ (*dotted line*), and $m = 7$ (*dash-dotted line*); (**b**) $m = 0$ (*solid line*), $m = 2$ (*dashed line*), $m = 4$ (*dotted line*), and $m = 6$ (*dash-dotted line*)

Here, in order to investigate the dependence of the Wigner function of the EGHZECSs on the parameters m and ϕ, we display the Wigner function as a function of ϕ with $|\alpha| = 0.3$, $|z_1| = 0.5$ and $|z_2| = |z_3| = 0.1$ for the odd number m in Fig. 4(a) and for the even number m in Fig. 4(b), respectively. The Wigner function depends on parameters m and ϕ as one can see clearly from Fig. 4. The negativity of the Wigner function indicates the EGHZECSs have nonclassical behavior of quantum states.

The CHSH inequality of (37) has 12 variable, and it is nontrivial to find the global maximum values of *BM* for all 12 variables. Fortunately, some local maximum values which violate the CHSH inequality can be found numerically using the method of steepest descent [61]. In Fig. 5 we plot the maximal Bell-violation as a function of ϕ for the different m, where $|z_1| = |z_2| = |z_3| = 0$, $|z_1'| = |z_2'| = |z_3'| = 0.002$ and $|\alpha| = 0.5$. We note that the maximal Bell-violation is also affected by the phase ϕ and excitation photon number m more signification. It is shown from Fig. 5 that the maximal Bell-violation increases with the parameter m in the case of the two sides of a periodicity ϕ while it decreases when ϕ lies on the middle range of a periodicity. In order to see clearly this characteristic, Fig. 6 is described as the maximal Bell-violation in the weak field ranges of $|\alpha|$ with $\phi = \pi/12$ for the different m, which shows that maximal Bell-violation significantly increases with the parameter m.

Fig. 5 Violation of the CHSH inequality for the EGHZECSs of a function of ϕ using parity measurements with $|z_1| = |z_2| = |z_3| = 0$, $|z'_1| = |z'_2| = |z'_3| = 0.002$ and $|\alpha| = 0.5$ for the different m as follows: $m = 0$ (*solid line*), $m = 1$ (*dashed line*), $m = 2$ (*dotted line*), and $m = 5$ (*dash-dotted line*)

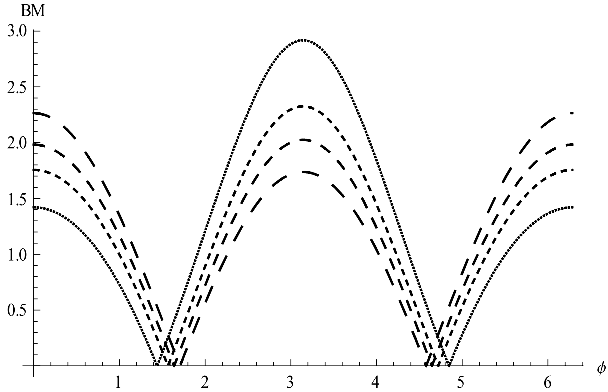
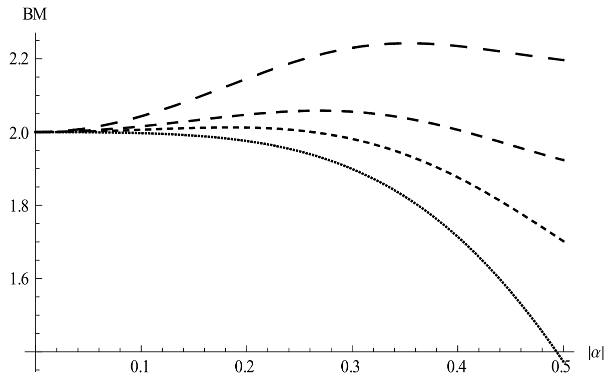


Fig. 6 Violation of the CHSH inequality for the EGHZECSs of amplitude α using parity measurements with $|z_1| = |z_2| = |z_3| = 0$, $|z'_1| = |z'_2| = |z'_3| = 0.002$ and $\phi = \pi/12$ for the different m as follows: $m = 0$ (*solid line*), $m = 1$ (*dashed line*), $m = 2$ (*dotted line*), and $m = 5$ (*dash-dotted line*)



5 Generation of the EGHZECSs

In the previous sections we have seen the important properties like entanglement and non-locality of the single-mode EGHZECSs. The question now arise of how such states can be generated in practice. In what follows, we propose two possible schemes, one is by using cavity QED and quantum measurement, another is by using BBO crystal and single-photon detection technique.

Firstly, we present a possible scheme, analogous to that proposed in [31], to generate them. The idea of the method is to prepare the electromagnetic field in the usual GHZ state and then to subject this state to interaction with a two-level atom in a resonator. Suppose that the two-level excited atom passes through the cavity. The atom makes a transition from the elicited state $|e\rangle$ to the ground state $|g\rangle$ by emitting a photon. The interaction Hamiltonian has the form ($\hbar = 1$)

$$H = g\sigma_+ a + g^* \sigma_- a^\dagger, \tag{47}$$

where g is the coupling constant, σ_+ and σ_- are the Pauli operator corresponding to the two-level atom, a and $a^\dagger$ are the Bosonic creation and annihilation operator of the field mode 1. Note that we can prepare the initial state of the field in the cavity in the form $|\phi(\alpha, 0)\rangle$ using, for example, the method discussed in [29, 30]. Then the initial state of the atom-field system is $|\phi(\alpha, 0)\rangle\ |e\rangle$. If the interaction time is sufficiently small, such that $gt \ll 1$, then

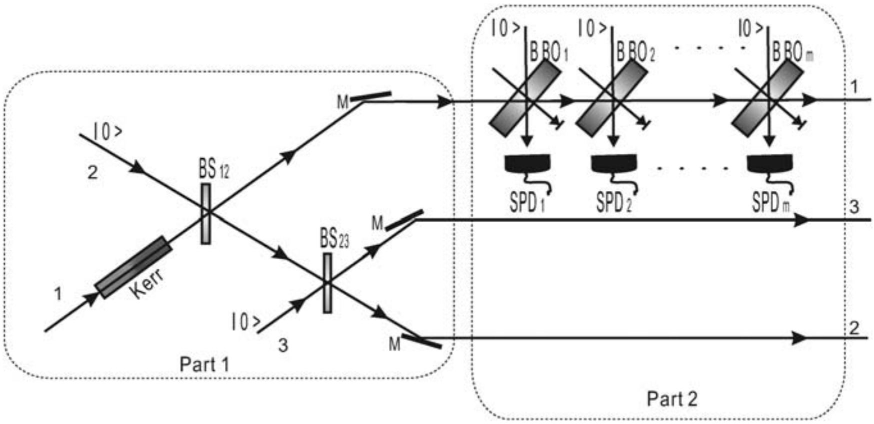


Fig. 7 Schematic setup of the generation of the single-mode EGHZECSs $|\phi(\alpha, m)\rangle$. Part 1 is composed of one Kerr medium, two beam splitters (BS) and three mirrors (M) and Part 2 contains m BBO crystals and m single photon detectors (SPD)

we expand the exponential to the first order and we can obtain the system state vector with the Hamiltonian in (47)

$$\begin{aligned} |\psi(t)\rangle &= \exp(-iHt)\,|\phi(\alpha,0)\rangle\,|e\rangle \\ &\simeq (1-iHt)\,|\phi(\alpha,0)\rangle\,|e\rangle \\ &= |\phi(\alpha,0)\rangle\,|e\rangle - ig^{*}ta^{\dagger}\,|\phi(\alpha,0)\rangle\,|g\rangle\,. \end{aligned} \tag{48}$$

Thus if the atom is detected in the ground state after it has passed through the cavity, then the state of the field is reduced to the single excited GHZ states with $m = 1$. If we consider a succession of m atoms through the cavity and if we detect all the atoms in the ground state $|g\rangle$, then the state of the two optical fields is in principle reduced to the desired state, the GHZ with m-photon excitations. In fact, the EGHZECSs can be generated in multiphoton emission processes, i.e., (47) is replaced by a new Hamiltonian with $a \to a^m$ and $a^\dagger \to a^{\dagger m}$

$$H' = g\sigma_+ a^m + g^*\sigma_- a^{\dagger m}, \tag{49}$$

the above procedure results in the field state $|\phi(\alpha, m)\rangle$.

Next based on the discussion, we propose another feasible scheme to optically generate the single-mode EGHZECSs. Our schematic setup is illustrated in Fig. 7, it can be divided into two parts: the generation of GHZ states by using the Kerr transformation and beam splitters (BSs) (Part 1) and m-photon excitations processes via the type-I BBO crystal and the single-photon detection (Part 2). Let us first consider part 1. The Hamiltonian of the nonlinear Kerr medium [19, 62] is $H_K = \chi(a^\dagger a)^2$, whose unitary evolution operator is

$$U_K = \exp\left[-i\chi(a^\dagger a)^2 t\right], \tag{50}$$

where χ is the nonlinearity coefficient, proportional to the nonlinear coefficient $\chi^{(3)}$ of the medium and interaction length. For the initial coherent state $|\alpha\rangle$, the state after the Kerr

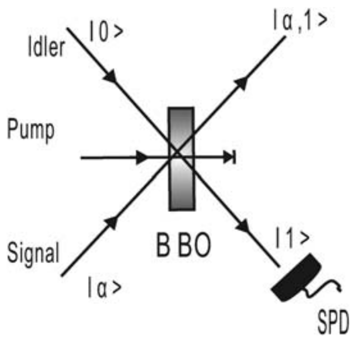


Fig. 8 Generation of entangle photon pairs in the oriented directions via the parametric down-conversion of pump light

interaction ($\chi = \pi/2$) is

$$U_K |\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\exp\left[-i\frac{\pi}{2}n^2\right](\alpha)^n}{\sqrt{n!}} |n\rangle$$
$$= \frac{1}{\sqrt{2}}\left(e^{-i\pi/4}|\alpha\rangle + e^{i\pi/4}|-\alpha\rangle\right). \tag{51}$$

The action of BS operator $B_{ij}(\theta) = \exp[\theta(a_i^\dagger a_j - a_j^\dagger a_i)]$ on two arbitrary modes i and j can be expressed as

$$B_{ij}(\theta)\, a_i^\dagger B_{ij}^{-1}(\theta) = a_i^\dagger \cos\theta - a_j^\dagger \sin\theta,$$
$$B_{ij}(\theta)\, a_j^\dagger B_{ij}^{-1}(\theta) = a_i^\dagger \sin\theta + a_j^\dagger \cos\theta, \tag{52}$$

where θ determines the reflectivity $r = \sin\theta$ and transmissivity $t = \cos\theta$ of BS. Combining the Kerr medium and two BSs in Fig. 7 (Part 1) and considering (51) and (52), we obtain the GHZ state $|\phi(\alpha, 0)\rangle$ as the output state of Part 1

$$|\phi(\alpha,0)\rangle = B_{23}\left(\sin^{-1}\frac{1}{\sqrt{2}}\right) B_{12}\left(\sin^{-1}\frac{1}{\sqrt{3}}\right) U_K \left|\sqrt{3}\alpha\right\rangle_1 |0\rangle_2 |0\rangle_3$$
$$= \frac{1}{\sqrt{2}}\left[e^{-i\pi/4}|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3 + e^{i\pi/4}|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3\right]. \tag{53}$$

In our scheme, the $|\phi(\alpha, 0)\rangle$ is used as the input state of Part 2. For the parametric down-conversion of the BBO crystal, one high-energy pump photon can annihilate into two photons of lower frequencies in symmetrically oriented directions being called the signal and idler modes. The Hamiltonian of the BBO crystal [36, 37, 63] is $H_C = \kappa(c^\dagger a_s a_i + a_s^\dagger a_i^\dagger c)$, here the operator c can be regarded as c-number $i\gamma$ for strong pumping, thus the corresponding evolution operator is

$$U_C = \exp\left[-i\kappa\left(c^\dagger a_s a_i + a_s^\dagger a_i^\dagger c\right)t\right] = \exp\left[\varepsilon\left(a_s^\dagger a_i^\dagger - a_s a_i\right)\right], \tag{54}$$

which is similar to the interaction with a two-level atom and the cavity in (47), where $\varepsilon = \kappa\gamma t$ may be regarded as an effective interaction time, $a_s^\dagger$ and $a_i^\dagger$ are the Bosonic creation operators of the sinal mode and idler mode, respectively. For an initial state $|\alpha\rangle_s|0\rangle_i$ (see

Fig. 8), as the parametric gain is sufficiently low, such that $|\varepsilon| \ll 1$, the form of output state can be written as

$$\begin{aligned} U_C \left|\alpha\right\rangle_s \left|0\right\rangle_i &= \exp\left[\varepsilon\left(a_s^\dagger a_i^\dagger - a_s a_i\right)\right] \left|\alpha\right\rangle_s \left|0\right\rangle_i \\ &\simeq \left|\alpha\right\rangle_s \left|0\right\rangle_i + \varepsilon a_s^\dagger a_i^\dagger \left|\alpha\right\rangle_s \left|0\right\rangle_i \\ &= \left|\alpha\right\rangle_s \left|0\right\rangle_i + \varepsilon' \left|\alpha, 1\right\rangle_s \left|1\right\rangle_i . \end{aligned} \tag{55}$$

When single photon is detected in the idler output mode, the correspond signal mode is involved in the stimulated emission of one photon, i.e., single-photon excited coherent state $|\alpha, 1\rangle$. Thereby, it is straightforward to design an array of BBO crystals in Part 2 in order to stimulate the desired multi-photon excitation upon the output signal mode. After successive elementary single photon excitation of the signal coherent field, the final output state can be obtained as

$$\begin{aligned} |\phi(\alpha, n)\rangle &= U_{Cm} \cdots U_{C2} U_{C1} \left|\phi(\alpha, 0)\right\rangle \\ &= \frac{1}{\sqrt{2}}\left[e^{-i\pi/4} \left|\alpha, n\right\rangle_1 \left|\alpha\right\rangle_2 \left|\alpha\right\rangle_3 + e^{i\pi/4} \left|-\alpha, n\right\rangle_1 \left|-\alpha\right\rangle_2 \left|-\alpha\right\rangle_3\right], \end{aligned} \tag{56}$$

under the condition that there have n SPDs which capture one photon, which is just single-mode n-photon ($n \leq m$) EGHZECSs.

For our present purpose, we may assume that the whole device is lossless and the effective interaction time of all BBO crystals need be just equal.

6 Conclusions

In summary, we have presented a class of the single-mode excited GHZ-type entangled coherent states (EGHZECSs), which are obtained through actions of creation operator on the GHZ-type ECSs and given their Schmidt decomposition in term of the excited even(odd) coherent state. Then we have exhibited the important properties such as entanglement and nonlocality of the single-mode EGHZECSs via investigating their concurrence of entanglement and examining their violation of CHSH inequality. It is found that these properties are affected by excitation photon number more signification. Finally, we have proposed two possible schemes to generate the single-mode EGHZECSs based on using cavity QED and quantum measurement and using BBO crystal and single-photon detection technique, respectively. It is mention that if we array multi-BBO crystals in each of the three output modes (see Part 1 of Fig. 7), three-mode EGHZECS is optically generated in the final output state. Thus we can expand excited three-mode GHZ-type ECS to multi-mode excited case and their properties can be discussed in the similar way as well.

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